OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 2640

Mechanics 4

Wednesday **21 JANUARY 2004** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF8)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- Where a numerical value for the acceleration due to gravity is needed, use $9.8\ \text{m}\,\text{s}^{-2}$.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 60.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages.

1 A wheel is rotating about a fixed axis, and is slowing down with constant angular deceleration $0.3\ \text{rad s}^{-2}$.

(i) Find the angle the wheel turns through as its angular speed changes from $8\ \text{rad s}^{-1}$ to $5\ \text{rad s}^{-1}$. [2]

(ii) Find the time taken for the wheel to make its final complete revolution before coming to rest. [3]

2 A rod AB of variable density has length 2 m. At a distance x metres from A, the rod has mass per unit length $(0.7 - 0.3x)\ \text{kg m}^{-1}$. Find the distance of the centre of mass of the rod from A. [5]

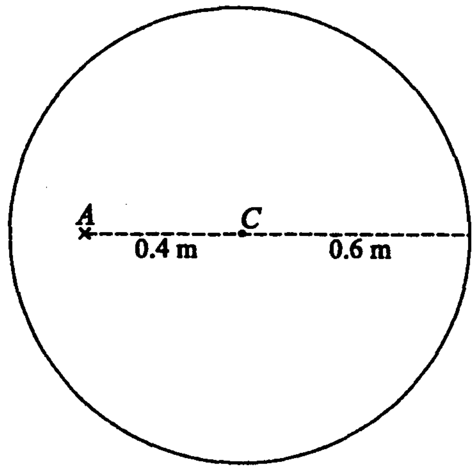
3 From a speedboat, a ship is sighted on a bearing of 045°. The ship has constant velocity $8\ \text{m s}^{-1}$ in the direction with bearing 120°. The speedboat travels in a straight line with constant speed $15\ \text{m s}^{-1}$ and intercepts the ship.

(i) Find the bearing of the course of the speedboat. [4]

(ii) Find the magnitude of the velocity of the ship relative to the speedboat. [3]

4 The region between the curve $y = \dfrac{x^2}{a}$ and the x-axis for $0 \leqslant x \leqslant a$ is occupied by a uniform lamina with mass m. Show that the moment of inertia of this lamina about the x-axis is $\frac{1}{7}ma^2$. [7]

5



A uniform circular disc has mass 4 kg, radius 0.6 m and centre C. The disc can rotate in a vertical plane about a fixed horizontal axis which is perpendicular to the disc and which passes through the point A on the disc, where $AC = 0.4$ m. A frictional couple of constant moment 4.8 N m opposes the motion. The disc is released from rest with AC horizontal (see diagram).

(i) Find the moment of inertia of the disc about the axis through A. [2]

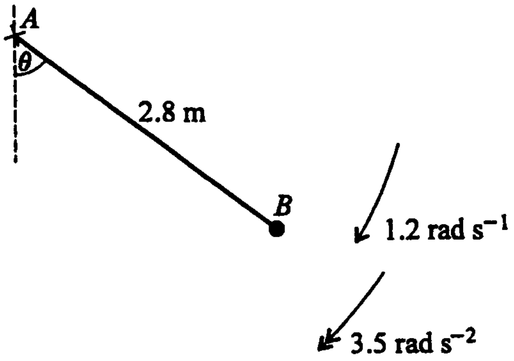
(ii) Find the angular acceleration of the disc immediately after it is released. [3]

(iii) Find the angular speed of the disc when C is first vertically below A. [4]

6 A rigid body consists of a uniform rod AB, of mass 15 kg and length 2.8 m, with a particle of mass 5 kg attached at B. The body rotates without resistance in a vertical plane about a fixed horizontal axis through A.

(i) Find the distance of the centre of mass of the body from A. [2]

(ii) Find the moment of inertia of the body about the axis. [2]



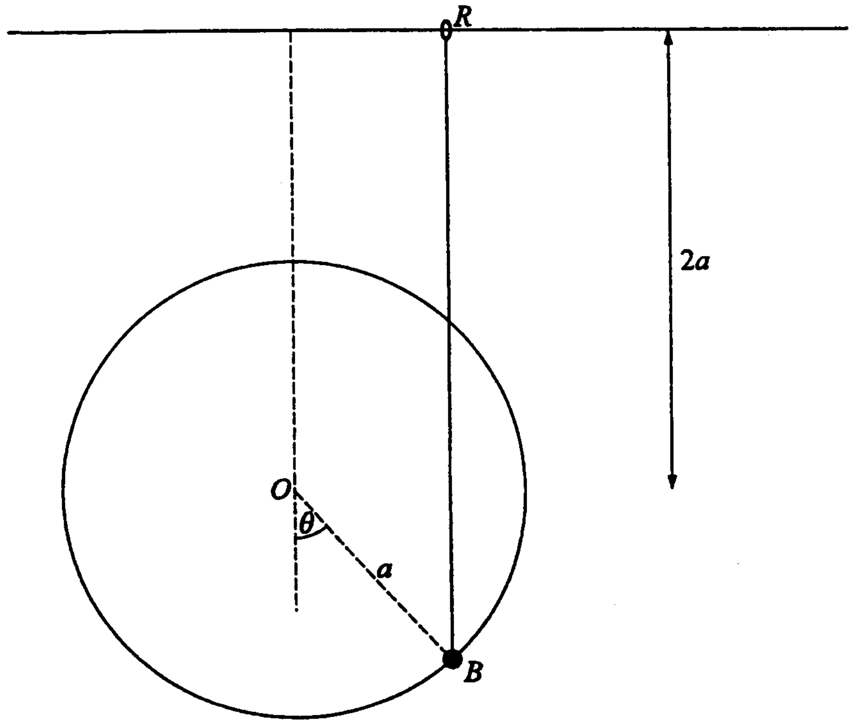
At one instant, AB makes an acute angle θ with the downward vertical, the angular speed of the body is $1.2\,\text{rad s}^{-1}$ and the angular acceleration of the body is $3.5\,\text{rad s}^{-2}$ (see diagram).

(iii) Show that $\sin\theta = 0.8$. [3]

(iv) Find the components, parallel and perpendicular to BA, of the force acting on the body at A. [6]

[Question 7 is printed overleaf.]

7



A small bead B, of mass m, slides on a smooth circular hoop of radius a and centre O which is fixed in a vertical plane. A light elastic string has natural length $2a$ and modulus of elasticity mg; one end is attached to B, and the other end is attached to a light ring R which slides along a smooth horizontal wire. The wire is in the same vertical plane as the hoop, and at a distance $2a$ above O. The elastic string BR is always vertical, and OB makes an angle θ with the downward vertical (see diagram).

(i) Show that $\theta = 0$ is a position of stable equilibrium. [7]

(ii) Find the approximate period of small oscillations about the equilibrium position $\theta = 0$. [7]

1

$\omega_1^2 = \omega_0^2 + 2\dot{\omega}\theta$ (equivalent of the linear $v^2 = u^2 + 2as$)

$8^2 = 5^2 + 2 \times {}^-0 \cdot 3 \times \theta$

$\theta = \mathbf{65}$ rad [2]

For the final complete revolution …

$\theta = \omega_1 t - \frac{1}{2}at^2$ ("$s = vt - \frac{1}{2}at^2$")

$2\pi = 0 - \frac{1}{2} \times {}^-0 \cdot 3 \times t^2$

$t^2 = 4\pi / 0 \cdot 3$

$t = 6 \cdot 4720... = \mathbf{6 \cdot 47\ s}$ [3]

2

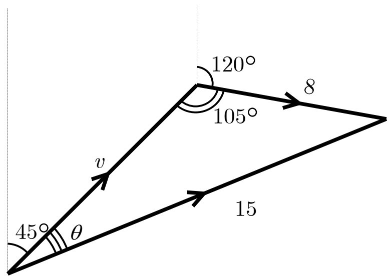
mass of rod $= \int_0^2 0 \cdot 7 - 0 \cdot 3x \; \mathrm{d}x = \left[0 \cdot 7x - 0 \cdot 15x^2\right]_0^2 = 0 \cdot 8$

$0 \cdot 8\bar{x} = \int_0^2 x\ 0 \cdot 7 - 0 \cdot 3x \; \mathrm{d}x = \left[0 \cdot 35x^2 - 0 \cdot 1x^3\right]_0^2 = 0 \cdot 6$

$\bar{\boldsymbol{x}} = \mathbf{0 \cdot 75}$ [5]

3

${}_{\text{speedboat}}\mathbf{v}_{\text{ship}}$ ${}_{\text{ship}}\mathbf{v}_{\text{sea}}$ ${}_{\text{speedboat}}\mathbf{v}_{\text{sea}}$



Sine Rule …

$\frac{\sin\theta}{8} = \frac{\sin 105}{15}$

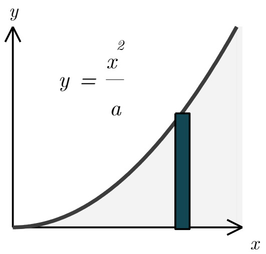
$\theta = 31 \cdot 008...$

So the **bearing of the speedboat** $= 45 + \theta = 076 \cdot 008... = \mathbf{076 \cdot 0}°$ [4]

$\frac{v}{\sin 75} = \frac{15}{\sin 105}$

$v = 10 \cdot 7858... = \mathbf{10 \cdot 8\ ms^{-1}}$ [3]

4



area of lamina $= \int_0^a x^2/a \; \mathrm{d}x = \frac{1}{3}a^2$

mass of 'elemental strip' $= \rho y \delta x = \frac{3m}{a^2} \cdot \frac{x^2}{a} \cdot \delta x$

M.o.I $= \int_0^a \frac{4}{3}\left(\frac{1}{2}y\right)^2 \cdot \frac{3mx^2}{a^3} \mathrm{d}x = \frac{m}{a^5}\int_0^a x^6 \, \mathrm{d}x = \frac{1}{7}\boldsymbol{ma^2}$ (show) [7]

5 $I_C = \frac{1}{2}mr^2 = \frac{1}{2}\times 4\times 0\cdot 6^2 = 0\cdot 72$

parallel axes theorem …

$I_A = I_c + m\times 0\cdot 4^2 = \mathbf{1\cdot 36\ \ kg\,m^2}$ [2]

at the instant of release …

$C = I\alpha$

$39\cdot 2\times 0\cdot 4 - 4\cdot 8 = 1\cdot 36\times\alpha$

$\alpha = \mathbf{8\cdot 00\ \ rad\,s^{-1}}$ [3]

energy considerations …

gain in K.E. = loss in G.P.E. - work done by friction

$\frac{1}{2}I\omega^2 = 4\times 9\cdot 8\times 0\cdot 4 - 4\cdot 8\times\frac{\pi}{2}$

$\omega^2 = 11\cdot 9708...$

$\omega = 3\cdot 45989... = \mathbf{3\cdot 46\ \ rad\,s^{-1}}$ [4]

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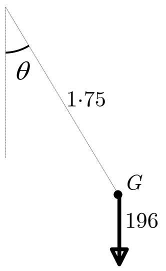
6 $M\bar{x} = \sum m_i x_i$ $\quad 20\bar{x} = 15\times 1\cdot 4 + 5\times 2\cdot 8$ $\quad \bar{\boldsymbol{x}} = \mathbf{1\cdot 75}$ [2]

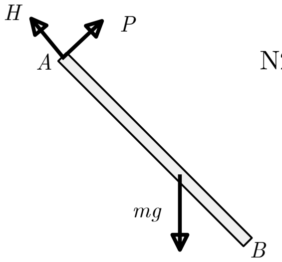
$I = \frac{4}{3}ml^2 + 5\,(2l)^2 = \frac{4}{3}\times 15\times 1\cdot 4^2 + 20\times 1\cdot 4^2 = \mathbf{78\cdot 4\ \ kg\,m^2}$ [2]



$C = I\alpha$

$196(1\cdot 75\sin\theta) = 78\cdot 4\times 3\cdot 5$

$\mathbf{sin}\,\theta = \mathbf{0\cdot 8}$ (show) [3]



N2 radially BA $\quad H - mg\cos\theta = m\omega^2 r$

$H = 20\times 9\cdot 8\times 0\cdot 6 + 20\times 1\cdot 2^2\times 1\cdot 75 = \mathbf{168\ N}$

N2 normal $\quad mg\sin\theta - P = mr\dot{\omega}$

$P = 20\times 9\cdot 8\times 0\cdot 8 - 20\times 1\cdot 75\times 3\cdot 5 = \mathbf{34\cdot 3}$ [6]

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7 Potential Energy Function (relative to position with $\theta = 0$)

[2]

$$V = mga\left(1 - \cos\theta\right) + \tfrac{1}{2}\cdot\tfrac{mg}{2a}\cdot\left(a\cos\theta\right)^2 - a^2$$

$$= \tfrac{1}{4}mga\left(\cos^2\theta - 4\cos\theta + 3\right) \qquad -\tfrac{\pi}{2} \le \theta \le \tfrac{\pi}{2}$$

$$\frac{dV}{d\theta} = \tfrac{1}{2}mga\left(-\cos\theta\sin\theta + 2\sin\theta\right) = \tfrac{1}{2}mga\sin\theta\left(2 - \cos\theta\right)$$

$$\left.\frac{dV}{d\theta}\right|_{\theta=0} = 0 \quad \text{and therefore } \theta = 0 \text{ is an equilibrium position.}$$

$$\frac{d^2V}{d\theta^2} = \tfrac{1}{2}mga\left(2\cos\theta + \sin^2\theta - \cos^2\theta\right) \qquad \left.\frac{d^2V}{d\theta^2}\right|_{\theta=0} = \tfrac{1}{2}mga > 0$$

So the potential energy function has a local **minimum** at $\theta = 0$ and therfore we have a position of **stable equilibrium**. [7]

conservation of mechanical energy …

$$\tfrac{1}{4}mga\left(\cos^2\theta - 4\cos\theta + 3\right) + \tfrac{1}{2}m\left(a\dot{\theta}\right)^2 = \text{constant}$$

differentiating …

$$\tfrac{1}{2}mga\dot{\theta}\left(2\sin\theta - \sin\theta\cos\theta\right) + ma^2\dot{\theta}\ddot{\theta} = 0$$

$$\ddot{\theta} = -\frac{g}{2a}\sin\theta\left(2 - \cos\theta\right)$$

for small oscillations …… $\sin\theta \approx \theta$ and $\cos\theta \approx 1$

and so …. $\ddot{\theta} \approx -\left(\dfrac{g}{2a}\right)\theta\,.$

The motion is therefore approximately SHM with period $T = \dfrac{2\pi}{\sqrt{g/2a}} = \mathbf{2}\pi\sqrt{\dfrac{\mathbf{2a}}{\mathbf{g}}}$

[7]